

S S DIVINE SCHOOL
Std 12 EM - Maths
ASSIGNMENT

Time :

Section A

- Write the answer of the following questions. [Each carries 2 Marks] [78]
1. Show that the relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.
 2. Give an example of a relation. Which is Symmetric but neither reflexive nor transitive.
 3. Check the injectivity and surjectivity of the following function : $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^2$
 4. Show that the Signum Function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by, $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$ is neither one-one nor onto.
 5. Let $f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof .
 6. Find gof and fog , if $f(x) = |x|$ and $g(x) = |5x - 2|$
 7. For given operation $*$ defined below, determine whether $*$ is binary, commutative or associative. On \mathbb{Q} , define $a * b = ab + 1$
 8. Find the value of the following : $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$
 9. Find the value of the following : $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$
 10. Find the principal value of the following : $3 \cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$
 11. Find the value of the following : If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x .
 12. Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by : $a_{ij} = \frac{(i+j)^2}{2}$
 13. Find the values of x, y and z from the given equation : $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$
 14. Compute the indicated product : $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \ 3 \ 4]$
 15. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that $(A + B)' = A' + B'$
 16. For the matrices A and B, verify that $(AB)' = B'A'$, where $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $B = [1 \ 5 \ 7]$
 17. Express the given matrix as the sum of a symmetric and a skew symmetric matrix : $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$
 18. Find area of the triangle with vertices at the point given in each of the following : (1, 0), (6, 0), (4, 3)
 19. Find value of k if area of triangle is 4 sq. units and vertices are : (-2, 0) (0, 4), (0, k)
 20. Using determinants find equation of line passing from point (3, 1) and (9, 3).

21. Write Minors and Cofactors of the element of following determinant : $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

22. Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

23. Examine the consistency of the system of linear equation in :

$$x + 2y = 2$$

$$2x + 3y = 3$$

24. Differentiate the functions with respect to x : $\sin(x^2 + 5)$

25. Differentiate the functions with respect to x : $\sec(\tan(\sqrt{x}))$

26. Find $\frac{dy}{dx}$ in the following : $2x + 3y = \sin y$

27. Differentiate the following w.r.t. x : e^{x^3}

28. Differentiate the following w.r.t. x : $\log(\cos e^x)$

29. Differentiate the function w.r.t. x : $\cos x \cdot \cos 2x \cdot \cos 3x$

30. Find $\frac{dy}{dx}$ of the function : $y^x = x^y$

31. Find the rate of change of the area of a circle with respect to its radius r when

(a) $r = 3$ cm

(b) $r = 4$ cm

32. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

33. Show that the function given by $f(x) = 3x + 17$ is increasing on \mathbb{R} .

34. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

35. Find the slope of the normal to the curve $x = a \cos^3\theta$, $y = a \sin^3\theta$ at $\theta = \frac{\pi}{4}$.

36. Integrate the function in Exercise : $\frac{2x}{1+x^2}$

37. Integrate the function in Exercise : $\frac{1}{x+x \log x}$

38. Integrate the functions : $\frac{3x^2}{x^6+1}$

39. Integrate the functions in exercise : $\sqrt{4-x^2}$

Section B

● Write the answer of the following questions. [Each carries 3 Marks]

[54]

40. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the X-axis in the first quadrant.

41. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the X-axis in the first quadrant.

42. Find the integrals of the functions : $\sin 3x \cos 4x$

43. Find the integrals of the functions : $\sin^4 x$

44. If x and y are connected parametrically by the equations, without eliminating the parameter, Find

$$\frac{dy}{dx} \cdot x = a(\cos\theta + \theta \sin\theta), y = a(\sin\theta - \theta \cos\theta)$$

45. If $y = 5 \cos x - 3 \sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

46. Find the second order derivative of the function : $x^3 \log x$

47. Using property of determinant prove that :
$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

48. Using the property of determinants and without expanding in following Prove that :

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

49. Using property of determinant prove that :
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

50. Using property of determinant prove that :
$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

51. Using elementary row transformations, find the inverse of each of the matrices : $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

52. Using elementary row transformations, find the inverse of each of the matrices : $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

53. Write the following function in the simplest form : $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$, $a > 0$; $\frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

54. Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}, \text{ for all } n \in \mathbb{N}.$$

State whether the function f is bijective. Justify your answer.

55. Find the integrals of the functions : $\sin^2(2x + 5)$

56. Find the integrals of the functions : $\sin x \sin 2x \sin 3x$

57. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

Section C

● Write the answer of the following questions. [Each carries 4 Marks]

[68]

58. Using elementary row transformations, find the inverse of each of the matrices : $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

59. Using elementary row transformations, find the inverse of each of the matrices : $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

60. If a , b and c are real numbers, and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$,

Show that either $a + b + c = 0$ or $a = b = c$.

61. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ then find $(AB)^{-1}$.

62. Solve system of linear equation, using matrix method :

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

63. Examine the consistency of the system of linear equation in :

$$x + y + z = 1$$

$$2x + 3y + 2z = 2$$

$$ax + ay + 2az = 4$$

64. Examine the consistency of the system of linear equation in :

$$2x - y = 5$$

$$x + y = 4$$

65. By using the properties of definite integrals evaluate the integrals : $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$

66. By using the properties of definite integrals evaluate the integrals : $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$

67. Evaluate the integrals : $\int_0^1 \frac{x}{x^2 + 1} dx$

68. Evaluate the integrals : $\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$

69. Evaluate the definite integral in exercise : $\int_{-1}^1 (x+1) dx$

70. Evaluate the definite integral in exercise : $\int_2^3 \frac{1}{x} dx$

71. Evaluate the definite integral in exercise : $\int_0^{\frac{\pi}{4}} \sin 2x dx$

72. Evaluate the definite integrals as limit of sums : $\int_a^b x dx$

73. Evaluate the definite integrals as limit of sums : $\int_1^4 (x^2 - x) dx$

74. Evaluate the definite integrals as limit of sums : $\int_0^2 (e^x - x) dx$